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Frequency Shifts Due to Ramsey Type Interrogation in Atomic Beam Tubes

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Frequency Shifts Due to Ramsey Type Interrogation in Atomic Beam Tubes

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FOREWORD

This Technical Note consists of two related papers on the topic of Frequency Shifts Due to Ramsey Type Interrogation in Atomic Beam Tubes. The first paper (Part I) was presented at the 25th Annual Symposium on Frequency Control, Fort Monmouth, N.J., April 1971; the second one (Part II) at the IEEE Eurocon, Lausanne, Switzerland, October 1971. This Technical Note may, therefore, be considered to be a documentation of these talks.

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PART I:

FREQUENCY BIASES IN A BEAM TUBE CAUSED BY RAMSEY EXCITATION PHASE DIFFERENCES

Helmut Hellwig, J. A. Barnes, and D. J. Glaze

A phase difference between the two interaction regions of a Ramsey excitation resonance structure results in a frequency bias in the measured beam resonance. A simple mathematical model is discussed which describes the dependence of this bias on the phase difference, the microwave power level, the modulation amplitude, and the resonance linewidth. This dependence results from the interaction of the modulated microwave excitation frequency with the asymmetric shape of the slightly shifted resonance line. In a first order approximation, no dependency on the frequency modulation amplitude is expected. Near-linear dependencies on the linewidth and microwave power level which are quite pronounced even at relatively small cavity phase differences are predicted.

The theoretical results are compared with one set of experimental data on the microwave power dependence as measured in 1969 with the cesium beam standard NBS-III. After a correction is applied to remove the power dependence due to spectral impurity of the microwave excitation, the remaining measured power dependence agrees quantitatively with that calculated using a cavity phase difference of about 3 milliradians.

Key words: Cavity phase shift; cesium beam tube; frequency accuracy; frequency standard; power shift; resonance line shape.

1. INTRODUCTION

In accordance with the international definition [1], the unit of time interval, the second, is realized with a cesium beam apparatus. Several bias corrections have to be applied to the measured resonance frequency in order to obtain the resonance frequency of the unperturbed, free cesium atom [2]. In this paper we consider only those bias corrections which are associated with a phase difference between the two interaction regions of a Ramsey excitation resonance structure [3].

In this paper we present a simple mathematical model which allows analytical solutions to the question of frequency shifts in the presence of a cavity phase difference. The results are intended to aid in the understanding of the physical mechanisms involved. Although our mathematical model is only an approximation to the physical situation, and is tailored to a particular set of conditions, we feel that it is close enough to the real conditions which were encountered in the National Bureau of Standards Frequency Standard [2], NBS-III, to warrant a quantitative comparison with experimental data from this apparatus. This is done in Section 5 of this paper.

2. CALCULATION OF FREQUENCY SHIFTS

We proceed on the basis of the following set of assumptions and approximations: (a) No frequency deviations from resonance exceed the linewidth; (b) the microwave interrogation power does not deviate greatly from optimum power;[1] (c) the modulation of the microwave signal is sinusoidal; (d) the modulation frequency is less than the resonance linewidth; (e) the velocity distribution in the atomic beam is Maxwellian; and (f) $\ell << L$ where ℓ and L are the lengths of the interaction and drift regions, respectively, as defined by Ramsey [3].

In the presence of a cavity phase difference the central peak of the Ramsey resonance pattern is displaced from the resonance frequency of the unperturbed atom. Furthermore, the lineshape displays a certain asymmetry. We restrict ourselves to the conditions (a) and (b) above and can in first order approximate the central peak of the Ramsey pattern, $I\{\nu\}$, by

---

[1] Optimum power is that microwave power which leads to a maximum in the detected beam intensity.

$$I\{\nu\} = A + B\varphi \sin\left\{\frac{\pi(\nu - \nu_\varphi)}{U}\right\} + C \cos\left\{\frac{\pi(\nu - \nu_\varphi)}{W}\right\} . \tag{I-1}$$

In eq (I-1) A, B, and C are constants under given experimental conditions; W is the linewidth at half intensity; U has the characteristic of a linewidth, $U \equiv W$ for monovelocity atoms, U is the same order of magnitude as W for our assumption (e) above; and ν_φ is a parameter related to the cavity phase difference φ. Using condition (e) above, it can be shown that

$$\nu_\varphi - \nu_o = \frac{3\varphi}{4\pi} W. \tag{I-2}$$

In eq (I-2) ν_o is the resonance frequency in the absence of a cavity phase difference. Because of the asymmetry, the frequency of the central peak ν_p differs from ν_φ. By setting $dI\{\nu\}/d\nu = 0$, we obtain

$$\nu_p = \nu_\varphi + \frac{B W^2 \varphi}{\pi C U} . \tag{I-3}$$

We now proceed to calculate the frequency to which the slave oscillator frequency servo will "tune-in." If we modulate ν in a sinusoidal fashion [condition (c) above] about some value ν_1 with the amplitude ν_m and the angular frequency ω we can write

$$\nu\{t\} = \nu_1 + \nu_m \sin \omega t . \tag{I-4}$$

We now make use of assumption (d) above and substitute eq (I-4) into eq (I-1), using eq (I-3) to express B. We obtain

$$\begin{aligned} I\{t\} = A + C \Bigg[(\nu_p - \nu_\varphi) \frac{\pi U}{W^2} \sin\left\{\frac{\pi}{U}(\nu_1 - \nu_\varphi + \nu_m \sin \omega t)\right\} \\ + \cos\left\{\frac{\pi}{W}(\nu_1 - \nu_\varphi + \nu_m \sin \omega t)\right\}\Bigg] . \end{aligned} \tag{I-5}$$

We are interested in the coefficient of that term in $I\{t\}$ which varies as $\sin \omega t$, because the servo forces this term to be zero.

Now

$$\sin\left\{\frac{\pi}{U}(\nu_1 - \nu_\varphi + \nu_m \sin \omega t)\right\}$$

$$= 2 \cos\left\{\frac{\pi}{U}(\nu_1 - \nu_\varphi)\right\} J_1\left\{\frac{\pi \nu_m}{U}\right\} \sin \omega t + \text{harmonics}$$

and

$$\cos\left\{\frac{\pi}{W}(\nu_1 - \nu_\varphi + \nu_m \sin \omega t)\right\}$$

$$= -2 \sin\left\{\frac{\pi}{W}(\nu_1 - \nu_\varphi)\right\} J_1\left\{\frac{\pi \nu_m}{W}\right\} \sin \omega t + \text{harmonics}.$$

Then the coefficient of $\sin \omega t$ in $I\{t\}$ [eq (I-5)] vanishes when

$$\sin \frac{\pi}{W}(\nu_1 - \nu_\varphi) = (\nu_p - \nu_\varphi) \frac{\pi U}{W^2} \cos \frac{\pi}{U}(\nu_1 - \nu_\varphi) \frac{J_1\left(\frac{\pi \nu_m}{U}\right)}{J_1\left(\frac{\pi \nu_m}{W}\right)} . \qquad \text{(I-6)}$$

ν_1 is the frequency to which the servo will "tune-in", and thus the resonance frequency of the beam tube. Note that ν_1 is not equal to either ν_p or ν_φ.

We have assumed that the offset is always small compared to the linewidth, i.e., $(\nu_1 - \nu_\varphi) << W$, $(\nu_1 - \nu_\varphi) << U$, and we can rewrite eq (I-6)

$$\nu_1 - \nu_\varphi \approx (\nu_p - \nu_\varphi) \frac{U J_1\left\{\frac{\pi}{U}\nu_m\right\}}{W J_1\left\{\frac{\pi}{W}\nu_m\right\}} . \qquad \text{(I-7)}$$

Substituting eq (I-2) and eq (I-3) into eq (I-7) we obtain

$$\nu_1 - \nu_o \approx \frac{3\varphi W}{4\pi}\left[1 + \frac{4B}{3C} \frac{J_1\left\{\frac{\pi}{U}\nu_m\right\}}{J_1\left\{\frac{\pi}{W}\nu_m\right\}}\right] . \qquad \text{(I-8)}$$

Equation (I-8) gives us the frequency bias $\Delta\nu$ which we define

$$\nu_1 - \nu_o \equiv \Delta\nu \,. \qquad \text{(I-9)}$$

In order to evaluate eq (I-8) we have to express B/C in terms of the operating conditions. Following eq (I-1) we choose a linear approximation based on the numerical results of [3], and we obtain $B\varphi\pi/CU \approx 2\varphi/3\pi W_o$. W_o is the linewidth at optimum power P_o. Equation (I-8) can now be written

$$\Delta\nu \approx \frac{3\varphi W}{4\pi}\left[1 + \frac{8U}{9\pi^2 W_o}\,\frac{J_1\left\{\frac{\pi}{U}\nu_m\right\}}{J_1\left\{\frac{\pi}{W}\nu_m\right\}}\right] . \qquad \text{(I-10)}$$

3. FREQUENCY MODULATION AMPLITUDE DEPENDENCE

In eq (I-10) any dependence on the frequency modulation amplitude is solely due to the arguments of the Bessel functions which contain ν_m. The approximate nature of our discussion requires that we restrict our discussion to modulation amplitudes not exceeding the optimum modulation, that is to

$$\nu_m \leq \nu_{mo} \text{ where } \nu_{mo} \equiv \frac{1}{2} W_o \,. \qquad \text{(I-11)}$$

This and the fact that U and W can be expected not to differ greatly leads us in a first order approximation to

$$\Delta\nu \approx \frac{3\varphi W}{4\pi}\left[1 + \frac{8W}{9\pi^2 W_o}\right] \qquad \text{(I-12)}$$

According to eq (I-12), $\Delta\nu$ has no functional dependence on ν_m. We conclude that (in the limits of our approximation) no frequency modulation amplitude dependent frequency bias is to be expected for $\nu_m \leq \nu_{mo}$. This result is in accordance with our experimental observations on the cesium beam frequency standards at the National Bureau of Standards.

4. POWER DEPENDENCE

We can use eq (I-12) to obtain the dependence of the frequency bias $\Delta\nu$ on the microwave power P. Equation (I-12) shows that the frequency bias is a function of the phase difference and the resonance linewidth. The linewidth, moreover, is a function of the microwave power level. This can be understood intuitively if one realizes that with decreasing microwave power the slower molecules become relatively more effective which results in a corresponding line-narrowing [3]. The frequency bias $\Delta\nu$ thus becomes power dependent. A numerical calculation of the relationship between linewidth and microwave power is depicted in figure 1. In figure 2 we depict the frequency bias $\Delta\nu$ as a function of both linewidth [eq (I-12)] and microwave power [eq (I-12) and fig. 1]. The ordinate is normalized to $\Delta\nu/\Delta\nu_o = 1$ at optimum power, $P = P_o$ and $W = W_o$, where $\Delta\nu_o$ denotes the frequency bias at optimum power.

Equation (I-12) and figure 2 show that the power dependence of the frequency bias is almost linear; however, we note that the extrapolation of this near-linear portion to $P \rightarrow 0$ does not yield a vanishing frequency bias. A reduction of power from P_o to $P_o/2$ will result in a frequency bias change of about 1/3 of the total bias at optimum power or more accurately

$$\Delta\nu\{P_o/2\} \approx 0.70\ \Delta\nu_o\ . \qquad \text{(I-13)}$$

5. EXPERIMENTAL RESULTS

The reported measurements were performed during the accuracy evaluation of the cesium beam frequency standard, NBS-III, in 1969 [2]. Figure 3 depicts the experimental results. The solid lines are a plot of the fraction frequency change as a function of the microwave power level for the two opposite beam directions. The precision of these measurements is discussed in [2]. The microwave power changes ranged from P (optimum power) to not less than one-fourth P_o. The actual value for P_o was

P_o = 1.4 mW; the frequency modulation amplitude was adjusted to $\nu_m \approx \nu_{mo}$. The two slopes are $+22 \times 10^{-13}$ per mW and -11×10^{-13} per mW. The total fractional frequency change with beam reversal at $P = P_o$ is 78×10^{-13} which gives us $\Delta\nu_o/\nu_o = 39 \times 10^{-13}$.

We believe that the asymmetry in the two slopes of figure 3 is caused by asymmetries in the spectrum of the microwave signal.[2] Such effects may be expected to be unchanged under beam reversal and to depend linearly on the microwave power (for a small single sideband perturbation) [3]. We obtain symmetric slopes if we subtract a slope of $+5.5 \times 10^{-13}$ per mW. Thus we can conclude that the spectrum causes a power shift of $+5.5 \times 10^{-13}$ per mW (dashed line in fig. 3), and the cavity phase shift a power dependence of $\pm 16.5 \times 10^{-13}$ per mW (dashed-dotted lines in fig. 3).

As we expected theoretically, we have a seemingly linear functional dependence. The measured biases at optimum power and half-optimum power are respectively, $\Delta\nu_o/\nu_o = 39 \times 10^{-13}$ and $\Delta\nu\{P_o/2\}/\nu_o = 27.5 \times 10^{-13}$ or

$$\Delta\nu\{P_o/2\} \approx 0.71\ \Delta\nu_o \qquad \text{(I-14)}$$

which is in agreement with the theoretical results of figure 2 and eq (I-13).

We can now calculate the cavity phase shift φ from eq (I-12). The numerical calculation yields

$$\Delta\nu_o = 0.26\ W_o\varphi. \qquad \text{(I-15)}$$

The linewidth was measured to be W_o = 45 Hz and we obtain from eq (I-15) with the power dependent frequency change discussed before $\varphi \approx 3 \times 10^{-3}$ radians.

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[2] The second-order Doppler effect is totally ignored in this paper; however, it must be taken into consideration when extreme accuracies are the objective. A. G Mungall recently treated this problem [4].

6. CONCLUSIONS

We presented an analytical discussion of frequency shifts in a beam tube which relate to the presence of a cavity phase difference between the two Ramsey excitation regions. The results are approximate and may be expected to be different for different experimental conditions (see Section 2).[3]

The results, as summarized in eqs (I-10) and (I-12) and in figure 2, show that frequency shifts caused by cavity phase differences, the microwave power level, the resonance linewidth (velocity of atoms), and the frequency modulation amplitude are closely interrelated.

Frequency changes due to changes in the microwave power level are quite pronounced, which is contrary to the result of a previous treatment of this subject [5]. In fact, the power dependence could be utilized to determine the frequency bias when other causes for frequency bias are known to be absent, e.g., microwave spectrum effects (see Section 5). In this case the measurement of the frequency change with reduction of the microwave power to, for example, $P_o/2$ yields directly the frequency bias according to eq (I-13). We have used this method in recent experiments on new cesium beam tubes.

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[3] The treatment of the power dependence due to cavity phase differences, as presented in this paper, leads to a rather simple physical picture: The chosen microwave power level acts like a velocity selector because only atoms within a rather narrow velocity range will have a significant transition probability. Atoms with velocities other than that selected by the given power level do contribute, but only in some minor fashion as may be seen from eq (I-12) and figures 1 and 2.

A cavity phase difference will, in general, also cause frequency shifts to occur if the frequency modulation amplitude is changed. The frequency changes are virtually absent if a frequency modulation amplitude of less than half the linewidth is chosen (Section 3). However, if larger frequency modulation amplitudes are used or if the cavity phase difference is unusually large (large bias) this effect could become significant. An analytical treatment which is valid for these conditions would require a higher order approximation than was attempted in this paper.

In the presence of a spectrum-related frequency bias of unknown magnitude, methods like the one discussed above will not be adequate. Beam reversal presently appears to be the only method[4] which then allows the separation and individual measurement of the different frequency biases. Beam reversal changes the sign in eq (I-12); consequently, one should then obtain microwave power (and frequency modulation amplitude) dependencies which are identical for the two beam directions, except for the sign reversal. Any deviations from this symmetry would indicate the presence of additional effects, e.g., signal spectrum asymmetry. The frequency biases can then be obtained as was demonstrated in Section 5 (fig. 3).

We summarize: In the absence of other biases, e.g., spectrum-related biases, variation of the microwave power offers a way to easily detect and correct for a cavity phase difference and the related frequency bias. This method requires a rather broad velocity distribution, e.g., a Maxwellian, and would not lead to any useful results in the limit of a monovelocity beam.

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[4] Beam velocity changes [6], as applied to the detection and correction of cavity phase differences, act in a way quite similar to the variation of the microwave power. Both act on W in eq(I-12). However, this method also should only be used with great caution if spectrum-related frequency biases are present.

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We thank Dr. Stephen Jarvis, Jr. for his assistance in computing the data of figure 1 and for his very valuable, constructive criticism.

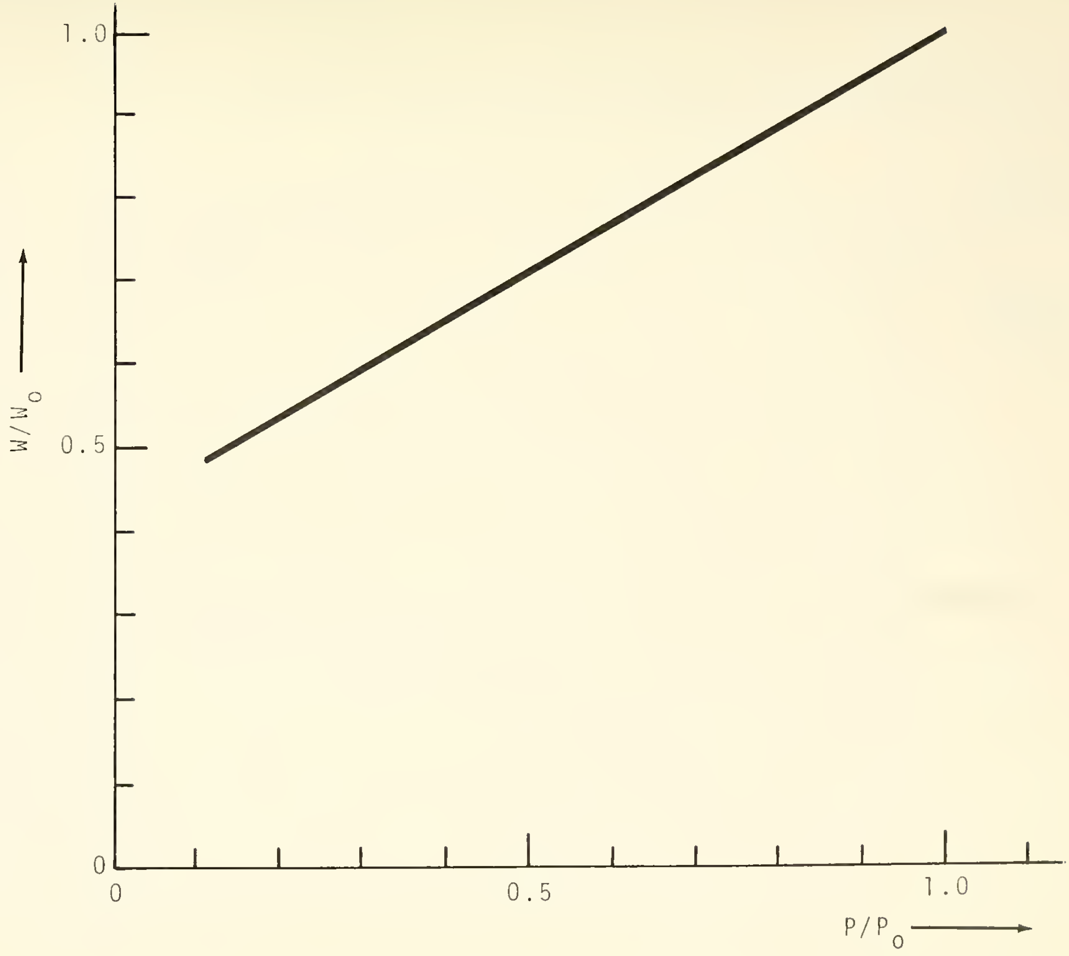

Fig. 1 Calculated dependence of the linewidth as a function of the microwave power. Linear approximation.

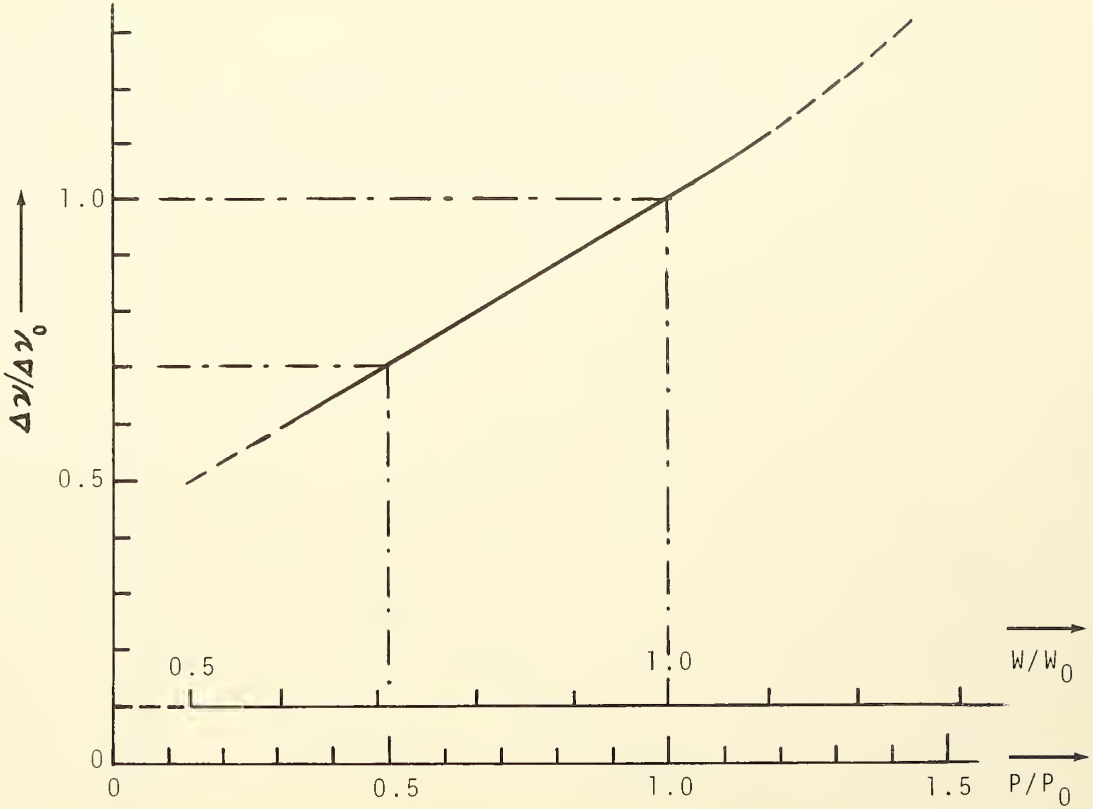

Fig. 2 Calculated frequency bias (normalized) as a function of linewidth and microwave power.

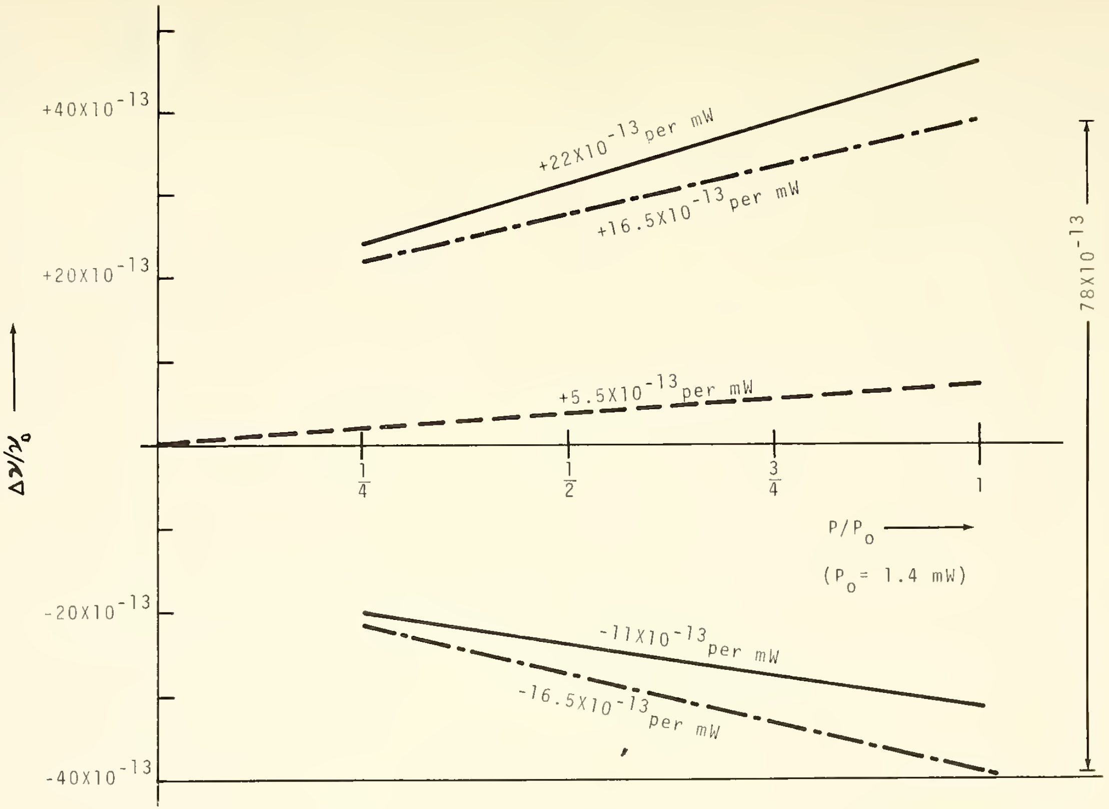

Fig. 3 Experimental results. Solid lines depict the fractional frequency shift measured as a function of the microwave power level (P_0 = 1.4 mW) for both directions of the atomic beam. The dashed line represents the bias attributed to spectral impurities of the microwave signal. The dashed-dotted lines are obtained by subtracting this bias from the measured results, and represent the bias due to a cavity phase difference. For details on the experimental procedure and a discussion of the measurement precision and accuracy, see [2].

PART II:

POWER-LINEWIDTH RELATIONS IN ATOMIC BEAM TUBES

Peter Kartaschoff

One of the most serious effects limiting the accuracy of atomic beam resonators is the uncertainty of phase shifts between the oscillating fields in a Ramsey excitation cavity. A technique using the linewidth dependence on the excitation power to measure the phase-shift induced bias has recently been proposed. The conditions under which this method is valid and their implications on the design of beam optics will be discussed with consideration of available experimental results.

Key words: Cavity phase shift; cesium beam tube; frequency accuracy; frequency standard; power shift; resonance line shape.

1. INTRODUCTION

One of the most serious effects limiting the accuracy of atomic beam resonators is the uncertainty of the phase difference between the two separated oscillating fields in a Ramsey cavity. A technique for measuring the frequency bias caused by this phase difference has been described by H. Hellwig, J. A. Barnes, and D. J. Glaze (Part I). This technique is based on the dependence of the resonance linewidth on the applied microwave power, which results in a power-dependent frequency shift if any phase difference is present. In this part, a different theoretical approach based on an approximation to the transition probability for single-velocity atoms leads to a simple relation between excitation power and linewidth which is valid for any sufficiently broad, but not necessarily Maxwellian, velocity distribution.

2. SINGLE VELOCITY POWER-LINEWIDTH RELATION

The theoretical considerations presented in Part I are valid only for a resonator where the full modified Maxwellian velocity as defined in ([3], eq II-26) is present at the detector. In many cases, however, especially in commercial-type beam tubes and in designs using multipole state-selectors, the velocity distribution of the detected atoms is severely restricted by the beam optics. This appears to be the result of ray-tracing optimizations attempting to maximize the "figure of merit" which is proportional to the ratio between the square root of the resonance amplitude and the linewidth.

In particular, a sharp cutoff in the velocity distribution is observed below the optimum velocity selected by the state selectors. In these cases, no significant narrowing of the linewidth is observed if the microwave excitation is reduced below the optimum value P_o.

On the other hand, there remains the possibility to broaden the linewidth by increasing the power beyond the optimum value provided that faster atoms can still be detected. Pending experimental verification, this technique appears feasible if the microwave signal is free from undesired sidebands.

The bias due to second order Doppler shift also depends on the velocity distribution and thus on the applied microwave power. This problem has recently been treated by A. G. Mungall [4].

For single velocity atoms, the transition probability given by Ramsey ([3], eq V-44) can be approximated near resonance in the following way, leading to his eq (V-44a):

$$P_{pq} = \frac{I}{I_{max}} = \frac{1}{2} \sin^2 2b\tau \, [1 + \cos(\lambda T + \varphi)], \qquad \text{(II-1)}$$

using Ramsey's notation, i.e.,

b is the perturbation amplitude in s^{-1}, as defined in [3];

$\tau = \frac{\ell}{v}$ the time of flight through each cavity;

$T = \frac{L}{v}$ the time of flight through the drift region between the 2 cavities;

$\varphi =$ the phase difference in radians between the oscillating fields;

$\lambda = \omega_o - \omega$ the angular frequency detuning, $\omega_o = 2\pi\nu_o$.

Note that I have changed the sign of φ with respect to that used by Ramsey, i.e., the first field is assumed leading the second. This has been done in order to remain consistent with Part I, where the same situation has been implicitly assumed.

Equation (II-1) is valid under the following assumptions:

$$2b >> |\lambda|$$

is the "near resonance condition" as stated by Ramsey.

It can be easily shown that this condition remains valid for a detuning larger than W/2 and also for a wide power range with a lower limit of about 0.3 P_o with $\ell \approx 1$ cm. The upper limit of applied power is only given by the validity of the perturbation method used in calculating the probability distribution and is thus beyond our considerations, since it requires $b << \omega_o$. We now compare eq (II-1) with eq (I-1). The linewidth being equal to $W = \frac{1}{T}$, the argument of the cosine term is

$$\lambda T + \varphi = \frac{\pi}{W}(\nu_o - \nu) + \varphi$$

and with $\varphi << 1$, we can write

$$P_{pq} = \frac{1}{2}\sin^2 2b\tau\,(1 + \varphi \sin\frac{\pi}{W}(\nu - \nu_o) + \cos\frac{\pi}{W}(\nu - \nu_o). \qquad \text{(II-2)}$$

Equation (I-1) reduces to the same form by setting there:

$$A = B = C = \frac{I_{max}}{2}\sin^2 2b\tau$$

$$U = W \text{ and } \nu_\varphi = \nu_o\,.$$

The peak frequency ($dI/d\nu = 0$) is then given by

$$\nu_p - \nu_o = \frac{W\omega}{\pi} \quad .$$

The "near resonance" condition assumed above replaces conditions (a), (b), and (f) and is less restrictive. Conditions (c) and (d) remain unchanged insofar as sinusoidal modulation is considered. (See Part I, Section 2.)

Condition (e) has no meaning here since we start from a single velocity approximation. From any known velocity distribution including the full modified Maxwellian, averaging and weighting can easily be done on a computer.

On the other hand, it is not possible to draw conclusions for a non-Maxwellian distribution from results assuming a full Maxwellian distribution. The usefulness of the latter assumption is therefore rather restricted. We therefore replace condition (e) by the following assumption: The velocity distribution function is assumed to be broad and to have bounded first and higher derivatives.

In other words, this means that we should be able to detect a significant number of atoms over a wide range, e.g., 2:1 velocity range, with intensity variations of not more than 2:1 over that range. Then we may consider the velocity selection being performed by the microwave power according to the factor $\sin^2 2b\tau$ in eq (II-1).

This factor has a maximum of unity for $b\tau = \pi/4$. Since the perturbation amplitude b is proportional to the square root of the applied power, the higher the applied microwave power, the higher is the velocity of the atoms for which the transition probability is maximum.

For the assumed fairly uniform velocity distribution the average velocity of the detected atoms is very close to the velocity where $\sin^2 b\tau$ is maximum and the linewidth can be calculated to a very good approximation by the relation

$$W = \frac{1}{2T_{av}} = \frac{v_{av}}{2L}$$

where T_{av} and v_{av} are the average time of flight and velocity, respectively. This is an approximation with an error of less than 3 percent even for the full modified Maxwellian, where

$$W = 0.65 \frac{\alpha}{L} = \frac{v_{av}}{2.05\ L} .$$

If we now call W_o the linewidth at the highest useful power P_o, the condition $b\tau = \pi/4$ = const. leads to the relation

$$\frac{W}{W_o} = \sqrt{\frac{P}{P_o}} .$$

Comparing this relation to figure 1 we see an exact fit at $P = P_o$ and $P = 0.5\ P_o$ and a very small error between these values. At very low power, our simple approximation is no longer valid.

Using eq (II-1) to calculate--as in Part I--the frequency ν_1 to which an electronic servo using sinusoidal modulation locks leads to the simple result:

$$\nu_1 - \nu_o = \nu_p - \nu_o = \frac{W\varphi}{\pi} .$$

That there is no dependence on modulation amplitude is trivial in our simple case because of the symmetry of the line approximated by eq (II-1). Hellwig, et al., have shown that there is no significant dependence even for the full modified Maxwellian distribution. The total shift due to the phase difference φ is 25% larger than that calculated for the full modified Maxwellian velocity distribution. This value appears to be more conservative for resonators having a restricted velocity distribution.

3. CONCLUSIONS

With the simple approximation of eq (II-1) derived directly from Ramsey's solution, we have thus been able to derive a relationship between power and linewidth which fits closely to the numerical data given in figure 1. We must point out again, however, that the expected variation of linewidth with excitation power is not observed in resonators having a narrow velocity distribution, or a sharp cutoff at the low velocity side of the distribution. There might be cases where there is a less sharp cutoff on the high velocity side and a correspondent line broadening could then be expected by applying higher than "optimum" power. Experimental investigation of this possibility is recommended.

Results obtained with a commercial cesium beam tube are shown in Table 1.

TABLE 1

P/P_o	$(P/P_o)^{\frac{1}{2}}$	W/W_o	I/I_{max}	$\sin^2 2b\tau$
0.5	0.707	0.971	0.80	0.80
1	1	1	1	1
2	1.414	1.17	0.638	0.632

These results are consistent with the assumption of a very narrow velocity distribution. The variation of the linewidth as a function of excitation power is much smaller than that expected for a wide or full modified Maxwellian velocity distribution. The resonance amplitude behaves almost exactly as if only one single velocity were present. This behavior is indeed very different from that reported on the NBS-III cesium beam resonator in Part I.

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**Frequency and Time Systems, Inc., Danvers, Massachusetts.

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

Part I: A phase difference between the two interaction regions of a Ramsey excitation resonance structure results in a frequency bias in the measured beam resonance. A simple mathematical model is discussed which describes the dependence of this bias on the phase difference, the microwave power level, the modulation amplitude, and the resonance linewidth. This dependence results from the interaction of the modulated microwave excitation frequency with the asymmetric shape of the slightly shifted resonance line. In a first order approximation, no dependency on the frequency modulation amplitude is expected. Near-linear dependencies on the linewidth and microwave power level which are quite pronounced even at relatively small cavity phase differences are predicted. The theoretical results are compared with one set of experimental data on the microwave power dependence as measured in 1969 with the primary cesium beam standard NBS-III. After a correction is applied to remove the power dependence due to spectral impurity of the microwave excitation, the remaining measured power dependence agrees quantitatively with that calculated using a cavity phase difference of about 3 milliradians.

Part II: One of the most serious effects limiting the accuracy of atomic beam resonators is the uncertainty of phase shifts between the oscillating fields in a Ramsey excitation cavity. A technique using the linewidth dependence on the excitation power to measure the phase-shift induced bias has recently been proposed. The conditions under which this method is valid and their implications on the design of beam optics will be discussed with consideration of available experimental results.

17. KEY WORDS (Alphabetical order, separated by semicolons)

Cavity phase shift; cesium beam tube; frequency accuracy; frequency standard; power shift; resonance line shape.

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